Aggregate and Regional Disaggregate Fluctuations

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ABSTRACT

This paper models fluctuations in regional disaggregates as a nonstationary, dynamically evolving distribution. Doing so enables study of the dynamics of aggregate fluctuations jointly with those of the rich cross-section of regional disaggregates. For the US, the leading state—regardless of which it happens to be—contains strong predictive power for aggregate fluctuations. This effect is difficult to understand if only aggregate disturbances affect aggregate business cycles through aggregate propagation mechanisms. Instead, a better picture might be one of a "wave" of regional dynamics, rippling across the national economy.

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1. Introduction

Macroeconomics, by definition, concerns aggregate economic variables. And, traditionally, macro empirics hews to this same discipline. In whichever mainstream version—real business cycle, aggregate demand and aggregate supply, or new Keynesian—theoretical and empirical macroeconomics studies the dynamic response of aggregate variables to hypothesized aggregate disturbances.

Departures from this focus exist, but are for the most part minor. In one instance, disaggregates are analyzed only to provide an aggregation theory, i.e., only to understand the macro implications of modelling the underlying micro units. The disaggregates themselves bear but auxiliary interest. In a second instance, the researcher might study empirically the behavior of consumers and firms, say in cross-section or panel data modelling, to understand their responses to changes in their environment. Often, the parameters of those disaggregates are then just presented as if immediately having implications for macroeconomic behavior. Such work views disaggregates as providing only *more* data (beyond aggregate time series), not *different* data. The latter, by contrast, is the view that this paper adopts.

There are, of course, counter-examples to the crude characterization just given. Interactions between individual income distributions and macroeconomic dynamics (e.g., Galor and Zeira [12] and Persson and Tabellini [19]), between relative prices and aggregate inflation (e.g., Lach and Tsiddon [15]), and between sectoral imbalance and aggregate unemployment (e.g., Evans [11] and Lilien [16]) are instances where disaggregate analysis has contributed insights for understanding macroeconomic fluctuations. In the same vein are the ideas that cross-sectional spillovers can cumulate for aggregate growth and fluctuations (e.g., Durlauf [8] and Long and Plosser [18]) and that gross labor flows—rather than just net ones—are informative for macroeconomic business cycles (e.g., Davis and Haltiwanger [7]).

All these counter-examples share an important distinctive feature. This is that there is significant *two-way* interaction between aggregate and disaggregate behavior: aggregates affect disaggregates, and disaggregates in turn affect aggregates.

Because the interaction is two-way, it contradicts the standard assumption, for instance, in panel data work where aggregate variables might affect disaggregates, but not vice versa. Moreover, as the income distribution and relative price examples make clear, the operative economic mechanism sometimes involves a relation between different parts of the disaggregates distribution: interaction between rich and poor, or tradeoffs between high- and low-priced commodities. Then, summary statistics of the distribution—say a conditional mean or cross-sectional variance—will be inappropriate for understanding the relation between disaggregates and aggregates.[1] What is needed, instead, is a way to analyze flexibly the dynamics of an entire distribution (or rich cross-section) of disaggregates.

Few econometric tools extant are appropriate for this. This paper seeks to add to those tools. It explores theoretical and empirical modelling of the joint dynamics of aggregate and regional disaggregate output. The regional disaggregates studied below—the states in the US—are large enough compared to aggregate US output that one cannot casually dismiss the potential effects of disaggregate dynamics on the aggregate. At the same time, there are many enough regional disaggregates to make apparent the modelling difficulties: standard vector time-series methods, for instance, will not do for modelling the dynamics of a 50 by 1 random vector.[2] If one were to turn then to the joint dynamics of European Union regional disaggregates—

[1] The easiest way to see this is through an example. Suppose that it is income inequality that matters for aggregate fluctuations and growth, as, e.g., in Galor and Zeira [12, 19] and Persson and Tabellini [12, 19]. Which income inequality measure should one use in empirical analysis? Theory doesn't always provide an answer since the simplified distributions that appear in a theoretical model are only suggestive of more general economic forces at work. Atkinson's classic paper [1] shows how alternative inequality measures imply substantively different—and potentially contradictory—views on the inequality actually extant.

[2] Post-War quarterly time-series now contain 200 observations. But a VAR model for a 50 by 1 vector already has 2500 free parameters in the first-order lag matrix coefficient; the variance-covariance matrix for the innovation contributes another 1275. Quah and Sargent [29] attempt to control this parameter prolif-

as in discussions of regional cohesion—one faces an 800 by 1 vector. Standard methods will be ill-suited for such analyses. This paper studies instead a technique to model the dynamics of a cross-section *distribution*. This technique, therefore, works regardless of how numerous the cross section units get.[3]

The empirical model of distribution dynamics developed below allows quantifying intra-distribution mobility, i.e., measuring how rapidly disaggregates traverse the cross-section distribution. Such measures provide natural calibrations of the speed of adjustment in a cross-section distribution to disaggregate perturbations. Seeing how those measures relate to movements in an aggregate (like GNP) reveals potential connections between aggregate fluctuations and gradual adjustments in disaggregates.[4]

The remainder of this paper is organized as follows. The next section sets down a simple, abstract theoretical framework for understanding the econometric calculations that follow. Section 3 presents some stylized facts; Section 4 gives more detailed analysis. Sections 3 and 4 are not, by any means, intended as formal statistical tests of the predictions in Section 2, only as groundwork for later, more complete study.

The key results from Section 4 are as follows. Mobility, while present, has little to do with aggregate fluctuations. However, leading states—which differ over time—have strong predictive power for aggregate output.[5]

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eration, using dynamic index representations, but for certain issues—identified below—those will be inappropriate.

[3] Nevertheless, because the cross-section disaggregates are studied in the form of their distribution dynamics, the framework necessarily cannot address every interesting question on diaggregate dynamics. For instance, spatial interaction is altogether ignored in the current work, although Quah [28] has used related techniques precisely to investigate such concerns.

[4] See, among others, Davis and Haltiwanger [7], Evans [11], Lilien [16], and Pissarides and McMaster [20].

[5] A referee has emphasized that the "leading states" findings are not special to

Finally, Section 5 briefly concludes.

2. A simple model

This section develops a simple theoretical model to analyze aggregate and regional disaggregate dynamics. The model is stylized to a degree where many interesting effects are absent, but in return it is explicit about the dynamics of the aggregate jointly with those of the entire disaggregate cross-section.[6]

One by-product of the reasoning below is to show the danger in interpreting as causal certain estimated relationships between aggregates and disaggregates. That, however, is not the main point of this section. Instead, the primary goal is to provide a theoretical framework for interpreting models of distribution dynamics.

To interpret variation along the time dimension, Sargent [30] has emphasized an optimizing, Euler equation characterization. Below, I do the same with variation over the cross section. Then, informally, I put the two together.

To focus on aggregate business cycles, assume that a single good is produced and consumed. To make different regions different, introduce a function $z(x)$,

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the new dynamic-distribution methodology developed in this paper. I agree, but have kept them in the paper nonetheless: those results do relate to the dynamic behavior of distributions, and they do serve to highlight how certain features of distribution dynamics are empirically important, and others not.

[6] In its focus on aggregate business cycles, the analysis here differs from economic geography work on location and agglomeration dynamics (e.g., Krugman and Venables [14]). In its focus on many, many regional disaggregates, it differs from that macro time series work (e.g., Engle and Kozicki [10] and Sargent and Sims [31]) which might seem pertinent and directly applicable; they are not, for reasons already given in the introduction. Work such as Barro and Sala-i-Martin [2], Blanchard and Katz [3], and Carlino and Mills [5]—which either average across the cross-section, or model regional disaggregates separately—are examples where one gets no information about the relation between different parts of the cross section of regions. Likely most directly relevant is work such as Ciccone and Hall [6], although the analyses there and in this paper differ substantially.

which maps location x to (productivity) characteristics z; the latter could be multi-dimensional, but is required to be non-negative in each entry. The analysis takes function z fixed, but eventually one would like to allow z to vary over time. Examples of z might include the work ethos on the Microsoft campus, Massachusetts's human capital in technology, and automotive engineering skills around Heathrow Airport: these all change through time in response to economic incentives.

Physical geography is a probability space $(\mathbb{X}, \mathcal{X}, \phi_x)$ that allows for possibly mixed discrete-continuous locations, nonuniform mountains, valleys, and plains, and so on. For different models, one might take $\mathbb{X}$ to be alternately a set comprising two points, a straight line, a circle, or a plane (finite or infinite). Then $\mathcal{X}$ comprises the collection of interesting subsets of $\mathbb{X}$, and the probability measure ϕ_x is a function from elements in $\mathcal{X}$ to $[0, 1]$; it evaluates members of $\mathcal{X}$ to measure their proportions out of total locations $\mathbb{X}$.

With this structure, z can also include measures of distance or accessibility of particular locations x. Physical distance, of course, doesn't change over time, but accessibility might, when roads and electronic highways are built. Therefore, function z can be viewed to have some components time-invariant and observable, others time-varying and unobservable, as well as combinations in between.

Denote employment at location x by $l(x)$; output is given by a standard neoclassical technology:

$$y(x) = f(l(x), z(x)), \tag{2.1}$$

assumed identical across locations. Assume that for any fixed z (including zero), the partial derivative $f_l = \partial f / \partial l$ diverges to infinity as l tends towards zero. In words, the first input of labor is always highly productive: for z unobservable, specifying f_l at zero z is only a normalization. A measure ϕ_x on locations, together with technology (2.1), a characteristics profile z, and an employment profile l, induces a measure each for characteristics ϕ_z, employment ϕ_l, and output ϕ_y.

The output profile y across locations, in turn, implies observed total output:

$$\overline{y} = \int y(x)\, \phi_x(dx) = \int f(l(x), z(x))\, \phi_x(dx).$$

Aggregate output $\overline{y}$ can always be calculated as above, mechanically, regardless of whether z and l are "good" or "bad" allocation profiles. Similarly, one can always find the distribution of wages across locations by calculating:

$$w(x) = f_l(l(x), z(x)) = \frac{\partial f}{\partial l}(l(x), z(x)),$$

again, mechanically.

More interesting is to ask whether particular allocations z and l, and their evolution over time, can be supported by some economic process. A useful starting point is to allow labor to move freely across locations. In that case, in equilibrium the labor allocation l adjusts so that wages equalize across x, and the labor market clears. Normalizing total labor supply to 1, this is:

$$\begin{aligned} f_l(l(x), z(x)) &= w(x) = \overline{w}, \\ \int l(x)\, \phi_x(dx) &= 1. \end{aligned} \tag{2.2}$$

A nonnegative function l solving (2.2) is a static (i.e., point-in-time) efficient labor allocation. Assuming f_l unbounded rules out corner solutions where some location might have zero employment. Given any distribution of characteristics ϕ_z, an allocation l satisfying (2.2) for some positive $\overline{w}$ is not just economically sensible; it turns out also to maximize aggregate output $\overline{y}$:

Proposition: *Denote by $\mathbb{M}_+$ the class of non-negative measurable functions on $(\mathbb{X}, \mathcal{X})$. Suppose that for x in $\mathbb{X}$ (a.e.-ϕ_x), the function $f(\cdot, z(x))$ has derivative $f_l = \partial f/\partial l > 0$, decreasing in l. Then the program*

$$\begin{aligned} \sup_{l \in \mathbb{M}_+} \quad & \int f(l(x), z(x))\, \phi_x(dx) \\ s.t. \quad & \int l(x)\, \phi_x(dx) \le 1 \end{aligned}$$

is solved by any l^* *in* $\mathbb{M}_+$ *such that there exists a positive number* ψ *for which simultaneously:*

(i) $f_l(l^*(x), z(x)) - \psi \leq 0$ (a.e. $-\ \phi_x$), *with equality whenever* $l^*(x) > 0$*; and*
(ii) $\int l^*(x)\phi_x(dx) = 1$.

(The proof of this result is in the technical appendix.)

Equation (2.2) implies that when $z(x)$ varies over x then so too will efficient employment levels. Stated this way, the result in the proposition seems trivial and obvious. However, the characterization also asserts that nothing about the cross-section standard deviation (or any other moment) of the observed distribution ϕ_l says anything about the behavior of total output about its maximum. There is a precise and natural economic relation embedded in (2.2), but it does not translate simply to, say, using dispersion as a measure of "imbalance" or "disequilibrium." To see this explicitly, it is useful to work through an explicit example.

Suppose z is scalar, and assume the technology:

$$f(l, z) = l^\alpha z^\beta, \qquad \text{for constants } \alpha \text{ in } (0,1) \text{ and } \beta > 0,$$

so that labor's marginal product is $w = f_l = \alpha l^{\alpha-1} z^\beta$. This implies local labor demand

$$l = (\alpha/w)^{1/(1-\alpha)} z^{\beta/(1-\alpha)}.$$

Labor market clearing then is

$$(\alpha/w)^{1/(1-\alpha)} \int z^{\beta/(1-\alpha)}\, \phi_z(dz) = 1. \tag{2.3}$$

Define an artificial random variable Z having the distribution given by ϕ_z. Using this notation the integral in the market-clearing condition (2.3) can be rewritten as the expectation $E\left(Z^{\beta/(1-\alpha)}\right)$. Then (2.3) implies the market clearing wage

$$\overline{w} = \alpha \left(EZ^{\beta/(1-\alpha)}\right)^{1-\alpha}. \tag{2.4}$$

Define $p(\alpha) = (1-\alpha)^{-1}$, and recall the definition of p-norm for a random variable,

$$\|Z\|_p = E\left(|Z|^p\right)^{1/p} \qquad \text{for } p \geq 1.$$

The market clearing wage (2.4) thus can also be written:

$$\overline{w} = \alpha \|Z^\beta\|_{p(\alpha)}.$$

Using this in the local labor demand function (2.3) we see that the equilibrium employment profile across regions is:

$$l(x) = (\alpha/\overline{w})^{p(\alpha)} z(x)^{\beta p(\alpha)}. \tag{2.5}$$

In characteristics z, employment is increasing, and either convex or concave depending on whether β is greater or less than $p(\alpha)^{-1} = 1 - \alpha$.

When all locations x have the same value of z, the measure ϕ_z places point mass on that value of z. The distribution of employment is then also degenerate at the value given by (2.5). In this special case, employment is equal across regions, and its being so happens to maximize aggregate output $\overline{y}$. In general, however, when z varies over x, then so too will l: equation (2.5) allows calculating ϕ_l from knowledge of ϕ_z. Aggregate output $\overline{y}$ in this more general case is no longer maximized by a degenerate distribution in l, or equivalently, by having zero (small) variance in employment across locations.

Regional output is given by substituting equilibrium employment into the technology:

$$\begin{aligned} y(x) &= (\alpha/\overline{w})^{\alpha p(\alpha)} z(x)^{\alpha\beta p(\alpha)} z(x)^\beta \\ &= (\alpha/\overline{w})^{\alpha p(\alpha)} z(x)^{\beta p(\alpha)} \end{aligned} \tag{2.6}$$

and therefore has the same shape in z as does employment. Despite curvature in the technology f, output y and employment l are, roughly speaking, collinear in equilibrium. But then the statements above on employment's distribution and its relation to aggregate output $\overline{y}$ apply immediately to the distribution ϕ_y of regional outputs and its relation to $\overline{y}$.

Integrating $y(x)$ across locations gives aggregate output. In our random variable notation, we can write this as:

$$\begin{aligned}\overline{y} &= (\alpha/\overline{w})^{\alpha p(\alpha)} E\left(Z^{\beta p(\alpha)}\right) \\ &= \|Z^{\beta}\|_{p(\alpha)}^{-\alpha p(\alpha)} \cdot \|Z^{\beta}\|_{p(\alpha)}^{p(\alpha)} \\ &= \|Z^{\beta}\|_{p(\alpha)} = \overline{w}/\alpha.\end{aligned}$$

Aggregate output therefore behaves as a particular absolute moment of the cross-section distribution of Z. In equilibrium, aggregate output will also be observed to move as does the real wage $\overline{w}$, only more so, since $|\alpha|$ is less than 1. Thus, when wages and output fluctuate over time, wages will be less variable than output.

Substituting this last relation between $\overline{w}$ and $\overline{y}$ into (2.6), we get:

$$y(x) = (\overline{y})^{-\alpha p(\alpha)} \cdot z(x)^{\beta p(\alpha)}$$

or

$$\log y(x) = (-\alpha p(\alpha)) \log \overline{y} + \beta p(\alpha) \log z(x). \tag{2.7}$$

Equation (2.7) looks like an (observable) index model representation for regional outputs: regional dynamics can be viewed as made up of a region-specific disturbance, $\beta p(\alpha) \times \log z(x)$, on top of some multiplier of aggregate output, $(-\alpha p(\alpha)) \times \log \overline{y}$. The development above, however, says that such a representation does *not* imply that aggregate output fluctuations "affect" regions with multiplier $-\alpha p(\alpha)$. Moreover, while the multiplier $-\alpha p(\alpha)$ depends only on parameters of the production technology (and is thus certainly policy-invariant and structural), there is no sense in which it describes the effect on regional outputs of aggregate output movements. Instead, regional and aggregate outputs are jointly determined: neither one causes the other.

Returning now to the general case, it is easy to see that much the same conclusions from the special case (except of course the precise functional form solutions) apply directly. Aggregate and local dynamics can be simply described: If the entire function z is perturbed, then a previously optimal employment profile l need no longer imply a uniform wage across regions, and labor will wish to reallocate towards a new profile. If we maintain the perfect mobility assumption, no new difficulties arise in the general case. We can take a random field—a doubly-indexed stochastic process—$z(x,t)$, where the t index denotes time, to drive the dynamics of the system. In each time period t, given $z(\cdot,t)$, the equilibrium employment profile $l(\cdot,t)$ again implies (i) a wage $\overline{w}(t)$ uniform across locations and (ii) maximized aggregate output $\overline{y}(t)$. There is also a resulting equilibrium profile of incomes across regions $y(\cdot,t)$: all the random fields, z, l, and y, indexed by x and t fluctuate across both space and time. Thus, embedded in the equilibrium is a sequence of evolving regional income distributions given by $\phi_y(t)$ fluctuating jointly with $\overline{y}$.

This discussion motivates a new view of aggregate and disaggregate disturbances. In the framework above, a natural definition of an aggregate disturbance is a perturbation that keeps the function z invariant in particular ways—for instance, aggregate disturbances might be associated with perturbations where z is simply shifted vertically. Disaggregate disturbances are then those perturbations that twist z's profile but maintain its vertical location. Put another way, disaggregate disturbances are a "wave" rippling through the surface comprising regional quantities.[7]

The model illustrates why certain kinds of empirical calculations are uninformative. For instance, from equation (2.7), stable relations between aggregates and measures of cross-section dispersion should not be viewed as movements in one implying movements in the other. Both the aggregate and the cross-section dis-

[7] Contrast this with a representation like that used in Quah and Sargent [29] where no similarly simple characterization is available. There, aggregate disturbances are characterized by an extensive set of dynamic orthogonality conditions.

tribution are jointly determined. Except in special cases, even the simplest theory gives neither a determinate sign on this relation nor the appropriate statistic of the cross-section distributions to look at.

The aggregate and disaggregate dynamics described thus far are simple and naive. Because I have assumed perfect mobility for labor, calculating the dynamic equilibrium path is easy: just string together in time the static equilibria across the cross-section distribution. Such analysis is interesting, however, for at least three reasons. First, it is convenient to calculate and easy to understand, and thus serves as a benchmark for more difficult calculations. Second, it makes explicit the limits on what we can hope to infer from data on disaggregate fluctuations: examining point-in-time statistics (like means, variances, modes, and so on) of the cross-section distributions is unlikely to be fruitful. Third, it suggests where further theoretical and empirical investigation can advance understanding.

To make progress beyond the simple model, some conjecture is needed on the economic mechanism underlying regional adjustments. One way to begin might be to make explicit the pattern of labor mobility, how readily one l profile translates into another. A useful story of regional fluctuations and aggregate business cycles, therefore, will likely embed within it the solution to the following abstract problem. How does a distribution of economic activity $\phi_{y,t}$ evolve—form the sequence of measures $\{\, \phi_{y,t+s},\ s = 1, 2, \ldots \}$—in response to an ongoing series of aggregate and idiosyncratic disturbances?[8]

What is needed, therefore, is a model that produces an equilibrium in the form of a vector stochastic difference equation in $\phi_{y,t}$, together with relevant aggregates: such an equation can then be estimated and simulated to give a characterization of steady states $\lim_{t\to\infty} \phi_{y,t}$. Tracing out the transition dynamics implied by that

[8] One reasonable guess is that in a model without externalities of the kinds in, e.g., Krugman and Venables [14], $\phi_{y,t}$ will, in response to a one time disturbance, converge to a distribution as characterized in the Proposition above. But, even if so, such "punctuated equilibrium dynamics" will never be observed in real world data since disturbances are ongoing through time.

equation then gives some insight into the interplay of fluctuations in aggregates and in the disaggregates dispersed over x.

3. Cross-section dynamics over one business cycle

We turn next to empirical application of the preceding ideas. To appreciate the issues to follow, begin by recording some facts on what happened over one complete NBER US business cycle upturn.[9]

Between 1982 and 1990—a complete NBER trough to peak span—average US per capita nominal personal income rose by 48%, an annual growth rate of 7%. Over this time, experiences across US states differed. Before looking at these, we establish preliminary intuition by considering some theoretical possibilities.

If the cross-section distribution about the average were stationary, then the relative positions of states might always remain unchanged: the richest states always remain richest; the poorest, poorest. Or, maintaining the hypothesis of an invariant cross-section distribution, some regional disaggregates might rise above the average from below, and others fall below the average from above. Stationary steady state, by itself, places no restriction on intra-distribution mobility.

If individual states were independent and identically distributed both in time and across each other, and the invariant cross-section distribution were symmetric, then any subset of states at the beginning of any time sample would have half end up below average and half above, by the end of the time sample. Since this holds

[9] The US states series here are constructed from Barro and Sala-i-Martin [2] and the Data Appendix in Blanchard and Katz [3]. As there, *state* refers to the 50 US states and the District of Columbia. I use nominal personal incomes throughout this section, rather than real. When there is a common general price index—because of the common national currency—the statements here carry over qualitatively unchanged to real personal incomes, as I am comparing behavior across states; similarly, if one studies personal incomes relative to *any* aggregate. The more formal analysis in the next section will use state incomes relative to per capita GDP.

for *any* subset of states, it must hold for those states beginning above average, as well as for those beginning below.

There is thus a range of possibilities, all consistent with well-behaved stationary fluctuations about an average (itself possibly varying through time).

We can take the "extreme-case" discussion further. Suppose only aggregate disturbances were important, and the propagation mechanism were an aggregate one. Then up to minimal variation, one should not expect significant asymmetries across states *over business cycles*. Different parts of the cross-section distribution should have roughly the same dynamic behavior relative to the aggregate and to each other. (The discussion in the rest of this section will not speak directly to this point, but we return to it in Section 4 below.)

What, in the event, transpired? The answer is none of the above. In 1982, at the beginning of the upturn, 20 states had per capita personal incomes above the US average. Over the upturn, one half of these—already relatively rich—saw their lead on the US average *increase*. Over the same upturn, again one half of the 31 states initially below average saw their relative incomes fall even further. Thus, the realized event differed from the hypothetical cases previously described: the cross-section spread apart over the upturn, and did so by systematically pulling out even further those parts of the distribution that were already at the extremes.

Details reinforce this conclusion. Of the 20 states that began above average in 1982, only three (Kansas, Texas, and Wyoming) transited below average by 1990; of the 31 initially below average, only one (Rhode Island) transited above average by 1990. Over this upturn the two fastest growers among states already richer than average were New Hampshire and New Jersey, whose leads on the average increased from 0.5% and 19% to 11% and 29% respectively. By contrast, the two worst growers among those already relatively poor were Louisiana and Oklahoma, whose income disparities from the average worsened from -12% and -1% to -26% and -19%, respectively.[10]

[10] Overall, the fastest grower was Maine (11% increase over 1982–90, from -19% to -8% about the US average); the worst, Alaska (-25% change over 1982–90),

What do these facts mean? Is it that as aggregate output grows over the business cycle, those states already rich gain most from the rise in aggregate activity? Or is it instead that rich states are the first ones affected by disturbances, and they then pull the nation out of recession? Whichever the reality is, these regional and state fluctuations are *large*. To see this, recall that most detrending techniques give business cycle fluctuations in US GNP of 3–4% about trend (see, among many others, Blanchard and Quah [4] and Prescott [21]).[11] If an average business cycle lasts 6 years, then the measured aggregate growth rate over an upturn is 4% over 18 months or 2.7% per year. From above, New Hampshire and New Jersey's fluctuations, about the national average, are close to 2% per year; those of Wyoming and Oklahoma, 3% per year. State fluctuations are thus large, compared to aggregate business cycles.

These stylized facts carry a meta-message, reinforcing statements made in the introduction. Intra-distribution dynamics contain regularities. However, the interesting regularities here will not be easily found using standard econometric techniques. To see this, recall what those techniques do. Estimating a panel data model on states or regions takes an average across the cross section (even when allowing for heterogeneities like "individual effects"). The differing behavior of states in the upper and lower parts of the cross-section distribution—if not averaged out exactly—will not be observable as starkly as described above. Certainly, how the top 10% of the distribution behaves relative to the bottom 10% is, in principle, available from panel data estimation (e.g., Lillard and Willis [17]); it is just that that kind of intra-distribution behavior is not obviously displayed there. Nor will intra-distribution dynamics be conveniently modelled.

By contrast, estimating individual time series models—one for each state, say, to permit differences across Maine and Oklahoma (e.g., Carlino and Mills [5])—leaves undetected the co-movements across states. The researcher then cannot

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Wyoming, and Oklahoma (both -18%).

[11] I take these numbers as a reasoned consensus although such numbers can, in theory, be dramatically altered by varying the detrending method (Quah [22]).

tell if rich states vary positively or negatively with poor ones. Attempting to model all those cross-sectional correlations leads, of course, to the (conceptual) degree-of-freedom problems already described in the introduction.

4. Distribution dynamics

This section develops an empirical model of aggregate and disaggregate fluctuations. The model is designed to take into account the issues previously described: it flexibly permits interactions between aggregates and disaggregates; it allows potentially many, many disaggregates; it captures how one part of the cross section distribution behaves relative to another. In brief, the model provides the law of motion for a sequence of dynamically evolving distributions.

Take the basic data to be the log of annual state personal income per capita relative to the national average each period.[12] Figure 4.1 is their three-dimensional plot; the states in this figure are arrayed in US Census ordering, i.e., beginning with Maine and Massachusetts, and ending with California, Alaska, and Hawaii. Such a graph emphasizes the data's rich variation across both time series and cross section dimensions. It clarifies why standard multiple time series modelling would be inappropriate here. The cross-section dimension in figure 4.1 has the same order of magnitude as does the time series: an investigator could not even estimate a full-rank variance-covariance matrix from these data, much less the dynamics in a multiple time series model of the 51-variate vector of state incomes.

[12] This choice is not inconsequential. If, at one extreme, the disaggregates were taken as the (fitted) idiosyncratic components from a dynamic observable index model, then by construction they would be everywhere orthogonal to the aggregate. If, at the other extreme, the disaggregates were unchanged from the original data, then purely mechanically some of their fluctuations would be the same as their aggregate's. The choice made in the text was for two reasons: first, convenience in interpretation; second, following, loosely, Section 2's discussion on "twists" of the cross-section distribution to define disaggregates.

Establish notation for the subsequent discussion: Let ζ be a fixed, finite-dimensional vector of aggregates—GNP growth rates, national unemployment, and so on—and y be a rich cross-section of disaggregates. The researcher seeks to characterize the dynamic evolution of the pair (ζ, y). The proposal here is simple: transform the system from (ζ, y) to (ζ, ϕ_y). (Recall from Section 2 that ϕ_y denotes the measure describing the cross-section distribution of y.) Then, the researcher models a system as depicted conceptually in figure 4.2.

The top panel of figure 4.2 contains the standard time-series plot of a scalar economic variable. The bottom panel plots the sequence of evolving distributions (implied by) $\{\phi_{y,t} : \text{ integer } t \geq 1\}$. It highlights two different characteristics in that sequence: (i) the changing shape of the distribution, and (ii) the intra-distribution dynamics, how a given part of the distribution at time t transits to another part of the distribution by time $t + s$. Call (i) and (ii) *shape* and *mobility* dynamics.[13] Subsequent analysis will decompose distribution dynamics into these two components.

To see how the decomposition works, recall that dynamically evolving probability measures $\{\phi_{y,t} : \text{ integer } t \geq 1\}$ can be written as a stochastic kernel equation:

$$\forall \text{ measurable } \mathcal{A} : \qquad \phi_{y,t+1}(\mathcal{A}) = \int \mathcal{M}_t(y, \mathcal{A})\, \phi_{y,t}(dy); \tag{4.1}$$

[13] Distinguish intra-distribution mobility (ii) from geographical mobility. The mobility here refers to moving about within the cross-section distribution of per capita incomes, not to moving about across physical space. In this scheme, if three states happen to have the same per capita income, they are not differentiated, even though two of them might be adjacent (like New York and New Jersey) and the third geographically distant (like California). Physical separation has little significance for the measures of economic activity studied here. For instance, when UPS relocated from Greenwich, Connecticut to Atlanta, Georgia, what mattered were the two locations' characteristics, *not* how far apart physically they happened to be (Financial Times, 28 October 1993). This idea on the irrelevance of physical separation is studied further in Quah [28].

(see, for instance, Stokey and Lucas [34, Ch. 8]). In general, ϕ_y might show more than first-order dependence. As with standard time-series state-space models, however, equation (4.1) is easily modified to permit that.

Each stochastic kernel $\mathcal{M}_t$ encodes information on both type (i) and (ii) dynamics in ϕ_y. When ϕ_y is discrete, then $\{\mathcal{M}_t : \text{integer } t\}$ is just a sequence of stochastic matrices (i.e., square arrays of non-negative numbers with row sums equal to 1). If, further, that sequence is time-invariant then (the observable) ϕ_y can be viewed as corresponding to the marginal distributions of an artificial (unobserved) Markov chain.[14] Then $\mathcal{M}_0 = \mathcal{M}_t$ (all $t \geq 1$) can be estimated directly from frequency counts.

When ϕ_y is continuous (or mixed discrete-continuous) then $\mathcal{M}_0$ can no longer be represented by a matrix, although it can still be analyzed using related methods (Quah [26]). If, however, $\mathcal{M}_t$ varies over time—as we wish to allow here—such analysis is no longer possible. Then, one way to proceed builds on the nonstationarities available in Markov-renewal structures; this is done in Quah [25], exploiting insights from Singer and Spilerman [33]. A second possibility is to decompose $\mathcal{M}$ explicitly into shape and mobility components—as suggested in figure 4.2—and then to parameterize their dynamics separately. We follow this approach here.

Fix a positive integer n: this will be the number of cells in a discretization of the basic data. Then, represent each stochastic kernel $\mathcal{M}_t$ by the pair $(M(t), q(t))$ where $M(t)$ is an $n \times n$ *fractile transition probability matrix* and $q(t)$ is an *n-element quantile set*, i.e., a collection of n disjoint random intervals. (A transition probability matrix is said to be *fractile* when it describes transitions out of cells containing equal fractions of the entire distribution.) To see that this gives a decomposition with the desired properties, it is easiest to provide constructive

[14] This reverses the usual reasoning where one observes the Markov chain, and then infers its unobserved associated probability distributions. Here, it is the cross-section distributions that are observed, and the Markov chain unobservable. Of course, the mathematics works the same either way.

definitions for M and q.

Denote the basic data by

$$\{\, y_j(t) : j = 1, 2, \ldots , N; t = 0, 1, \ldots , T \,\}$$

where j denotes cross-sectional units and t indexes time. The sequence ϕ_y relates to the basic data by

$$\forall r \in \mathbb{R} : \qquad \phi_{y,t}\big((-\infty, r]\big) = \#\,\{\, j : y_j(t) \leq r \,\} \times N^{-1}.$$

Every fixed positive integer n implies a unique set of equally-spaced probabilities

$$\{\, m/n : m = 0, 1, \ldots , n \,\}.$$

Define at time t the quantiles

$$(quant)_m(t) = \inf \{\, r \in \mathbb{R} \mid \phi_{y,t}\big((-\infty, r]\big) > m/n \,\}, \qquad m = 1, 2, \ldots , n;$$

and take

$$(quant)_0(t) = -\infty.$$

These give the consecutive disjoint random intervals

$$q_m(t) = \big((quant)_{m-1}(t), (quant)_m(t)\big], \qquad m = 1, 2, \ldots , n,$$

which, in turn, comprise the quantile set

$$q(t) = \{\, q_m(t) : m = 1, 2, \ldots , n \,\}.$$

By construction, $\phi_{y,t}(q_1(t)) = \phi_{y,t}(q_m(t))$ for all m, i.e., the elements of every quantile set have equal measure.

The sequence of quantile sets together with the basic data define the transition probabilities M: let matrix $M(t)$ have (l, m) entry

$$M_{lm}(t) = \frac{\#\{\, j : y_j(t+1) \in q_m(t+1) \text{ and } y_j(t) \in q_l(t) \,\}}{\phi_{y,t}(q_l(t))}, \qquad l, m = 1, 2, \ldots, n.$$

Clearly, $M(t)$ comprises all non-negative entries and has row sums equal to 1. Also immediate, by construction, is that each $M(t)$ is fractile, i.e.,

$$\left(\sum_{m=1}^{n} M_{lm}(t)\right)\phi_{y,t}(q_l(t)) = \phi_{y,t}(q_l(t)) = \phi_{y,t}(q_1(t))$$

is the same for all l.

To summarize, M encodes information on mobility while q encodes information on shape. We can further clarify M's role by using a *mobility index* (Geweke, Marshall, and Zarkin [13] or Shorrocks [32]). Analogous to measures of income inequality—summarizing the information in an entire distribution into a single scalar—a mobility index collapses into one number the mobility information in a transition probability. However, as already emphasized above for inequality measures, no single mobility index need be completely satisfactory. Thus we consider four such indexes (three from Geweke, Marshall, and Zarkin [13] and Shorrocks [32], and one new). The stochastic kernel representation $(M(t), q(t))$—a pair for each time period—will imply time series on each of these indexes.

First, take Shorrocks's index μ_1 defined by:

$$\begin{aligned}\mu_1(M) &= \frac{n - \mathrm{tr}(M)}{n-1} \\ &= \left(\frac{n}{n-1}\right)\left\{n^{-1}\sum_j (1 - M_{jj})\right\},\end{aligned}$$

where M_{jj} denotes the j-th diagonal entry of the matrix M. Since $1 - M_{jj}$ is the probability of exiting state j, Shorrocks's μ_1 is the inverse of the harmonic mean of expected durations of remaining in a given part of the cross-section distribution. It thus provides one natural index of mobility: the higher is $\mu_1(M)$, the less "persistence" is there in M.

Since the trace of a matrix equals the sum of its eigenvalues, Shorrocks's index can also be written as:

$$\mu_1(M) = \frac{n - \sum_j \lambda_j}{n-1},$$

where λ_j are the eigenvalues of M. Thus when M's eigenvalues are all real and non-negative, Shorrocks's μ_1 is identical to the second index we consider:

$$\mu_2(M) = \frac{n - \sum_j |\lambda_j|}{n-1};$$

in general, however, μ_1 and μ_2 will differ.

To see the motivation behind μ_2 recall that every stochastic matrix M always has one eigenvalue equal to unity, and all its other eigenvalues bounded from above by 1 in modulus. In the most regular case, when M implies a unique ergodic distribution, the sequence $\{ M^k : k \geq 1 \}$ converges to that distinguished matrix having all rows equal to the ergodic distribution.[15] Convergence occurs at a geometric rate, given by powers of (Jordan blocks in) the eigenvalues λ_j. Thus the smaller is the modulus of an eigenvalue—the larger is $1 - |\lambda_j|$—the faster does the corresponding component in M^k converge. Putting these facts together, we see that μ_2 sensibly indexes mobility; it relates positively to the average rate of convergence of the cross-section distribution towards the ergodic limit.

When all eigenvalues except the unit one are strictly less than 1 in modulus, then as horizon k grows, the dominant convergence term is given by $|\lambda_2|$, the modulus of the second largest eigenvalue. Thus, for the same reason that μ_2 is sensible, one might consider our third mobility index:

$$\mu_3(M) = 1 - |\lambda_2|.$$

[15] When M is fractile—as here—the uniform distribution is always an ergodic limit.

This, like μ_2, indexes the speed of convergence. But whereas μ_2 incorporates *all* the different rates of convergence, μ_3 captures only the asymptotic rate. The two, μ_2 and μ_3, would be identical—up to a scaling involving only n—when evaluated at an M whose smallest eigenvalues, beyond the largest two, turn out to be zero.

Mobility indexes μ_1, μ_2, and μ_3 have previously appeared (Geweke, Marshall, and Zarkin [13] and Shorrocks [32]). The discussion above gives conditions under which they coincide, but in general they are not directly related to each other. These mobility indexes—like all others I know in the literature—use only transition probability M information. But the quantile sets also contain relevant information. It means one thing to transit from the poorest 10% to the richest 10% when the poor and rich differ by only a small amount; it means something else when the poor and rich are orders of magnitude apart. Thus for the last mobility index considered here, I bring in information on the quantile sets.[16]

To motivate this new index, denoted μ_{AR}, note that the evolution of $\phi_{y,t}$ (and thus of the stochastic kernel $\mathcal{M}_t$) implies an unobservable *scalar* stochastic process $\{\widetilde{y}_t : \text{ integer } t\}$. If for each t, the artificial variable $\widetilde{y}_t$ has finite variance, then—even though $\widetilde{y}$ is *never* observed—one can calculate the projection $P[\,\widetilde{y}_{t+1} \mid \widetilde{y}_t\,]$ from knowledge of just ϕ_y and $\mathcal{M}$.

(I say that $\widetilde{y}$ is artificial because it is not observed directly but only hypothesized by the researcher. In standard time-series analysis, a researcher uses observations on a scalar (or vector) time series Y and then hypothesizes a distribution for that random variable (this is particularly apparent in, say, ARCH models). Here, the opposite happens: the researcher observes the empirical (cross-section) distribution and then hypothesizes a random variable to go along with that distribution.

[16] Yet another mobility index that might be considered measures the speed of transition across the support of the evolving distributions. In its simplest form, it could be the mean first-passage time from the bottom to top 10% of the distribution (as used for different purposes in Durlauf and Johnson [9] and Quah [26]). However, experimentation showed such indexes uninformative here; I thus omit the results using them.

As Quah [24, 26] has observed, this is usefully viewed as the dual to standard practice in time-series analysis. To compute the projection $P[\,\widetilde{y}_{t+1} \mid \widetilde{y}_t\,]$, one never needs to observe $\widetilde{y}$; only the sequence of dynamic cross-moments is needed, and ϕ_y and $\mathcal{M}$ together readily allow calculating that.)

Call ρ_t the coefficient on $\widetilde{y}_t$ in this projection, and define the mobility index

$$\mu_{AR,t} = \mu_{AR}(M(t), q(t), q(t+1)) = 1 - \rho_t.$$

Call this our fourth mobility index. Why does it measure mobility? Some special cases help answer this. When M is the identity matrix, there is extreme persistence. All parts of the cross-section distribution remain exactly where they begin. When, further, $\phi_{y,t} = \phi_{y,t+1}$, then μ_{AR} is easily shown to be 0. Maintaining this assumption for ϕ_y, suppose now that M only has 1's on the main anti-diagonal: the richest become the poorest; conversely the poorest, richest. A simple calculation then gives μ_{AR} equal to 2 for this case of "extreme mobility" (unlike μ_1, μ_2, and μ_3 the new index μ_{AR} is not restricted to lie between 0 and 1; a simple transformation can, of course, enforce that restriction although nothing relevant hinges on it).

Keeping the skew-symmetric M, now let $\phi_{y,t+1}$ be a mean-preserving spread on $\phi_{y,t}$. The richest now become even poorer than the originally poorest, and vice versa. There is thus greater mobility than before. The value of μ_{AR} increases above 2; by contrast, any mobility index defined only on M would remain unchanged.

Using the same reasoning, when $\phi_{y,t+1}$ is a mean-preserving spread on $\phi_{y,t}$, but M is the identity matrix, the index μ_{AR} decreases below 0. Again, any mobility index ignoring q would remain invariant—concealing that in this example the rich have become richer and the poor poorer.[17] Thus, μ_{AR} correctly captures our intuition on intra-distribution mobility.

[17] There are, of course, situations where ignoring ϕ_y is appropriate. If M has all rows identical (and thus equal to the ergodic distribution), the index μ_{AR} equals 1, independent of ϕ_y. In this case, the artificial $\widetilde{y}$ is independent (but not necessarily

Figure 4.3 plots, in its first panel, aggregate GNP annual growth rates, and in subsequent panels μ_1, μ_2, μ_3, and μ_{AR}, respectively. The mobility indexes fluctuate; they are neither extremely persistent nor serially independent. There are periods—for instance the mid-50s—when all the indexes are large together, but also periods—for instance the 80s—where they show little co-movement.

Over the entire sample the mobility indexes all correlate positively with each other. Thus, even if no single index is perfect, at least all show the same tendencies. The largest correlation is 0.41 for μ_2 with μ_3, suggesting that even for short-run dynamics higher-order mobility and distribution convergence rates are negligible. The smallest correlation is 0.15 for μ_3 with μ_{AR}, suggesting that the location content in μ_{AR} does contain potentially important, independent information.

However, that independent information in μ_{AR} turns out not to be important for aggregate fluctuations. The contemporaneous correlation of μ_{AR} with aggregate growth rates is just 0.06, compared with 0.15 for μ_2 with aggregate growth rates, and -0.24 for μ_1 (for μ_3 it is 0.02). Thus, the evidence is weak that intra-distribution mobility has much to do with aggregate GNP movements. The suggestive evidence over a single upturn, previously discussed in Section 3, turns out to be no more than suggestive. The conjectured links there have no firm basis over the entire sample.[18]

Turn now to shape dynamics. Quah [23] provides Granger-causality calculations suggesting that it is median and the maximum of the distribution of US states' relative incomes that are most strongly dynamically correlated with aggregate growth rates. Roughly speaking, the maximum Granger-causes aggregate output (but not vice versa), while aggregate output Granger-causes the median

identically distributed) in time. It is appropriate that this case is intermediate between those discussed in the text.

[18] Might dynamic correlations overturn this conclusion? The answer is no. Computing bivariate vector autoregressions in aggregates and mobility indexes showed no interesting significant patterns of Granger causality. Quah [23, 27] discusses other properties of the transition probabilities M.

(again, but not vice versa). Here, I report results from extending that analysis.

The table gives a compact description of tests of (Granger-causality) exclusion restrictions in trivariate VARs in aggregate growth, the median of the cross section income distribution (in levels), and the maximum (again of the distribution in levels). Results are presented here for 2- and 5-lag VARs, extremes in the range I tried: larger systems have too few degrees of freedom to say anything precise; systems in between have results intermediate between the extremes given. In the table each cell entry is a pair of numbers giving marginal significance levels for excluding that right-hand-side bloc from the named left-hand-side variable. The first number in each cell is the marginal significance level in the 2-lag VAR; the second, the 5-lag.

The maximum's strong predictive content for aggregate growth rates manifests once again. The marginal significance level for excluding the maximum in the equation for aggregate growth is between 3% and 4%, and is the smallest of the table's off-diagonal entries. As pointed out in Quah [23] it is *not* that any single state or region is responsible for this predictive power. Five different states, at different times, were at this point of the distribution. Connecticut was there the longest (17 non-consecutive years out of 43), but using it in the VAR in place of the maximum loses all predictive content: marginal significance levels increased to over 75% in all cases.

The maximum, however, does not help to predict the median: marginal significance levels here exceed 40%. In shorter-lag systems, the median appears to help predict the maximum; however, that predictive power is unstable, and vanishes once longer-lag systems are considered. Unlike in bivariate systems the aggregate no longer helps to predict the median—again, marginal significance levels for excluding aggregates from the median's equation exceed 20%.

To conclude, the maximum of the distribution—the leading state—contains important predictive information for the aggregate. Little else is stable and significant. These conclusions are difficult to understand if one views aggregate fluctuations as aggregate disturbances moving through an aggregate propagation mechanism. Under that scenario, all parts of the distribution should behave symmetrically with the aggregate; there is no reason why different disaggregates should behave systematically differently in relation to aggregate fluctuations.

Instead, the results here suggest a different picture. Asymmetry is important across different portions of the cross section distribution. Fluctuations appear to constitute a "wave" rippling across regions; the initial impulse for that wave varies, but, on average, locates in the highest-income states.

5. Conclusions and extensions

This paper has provided a framework for analyzing comovements in aggregate and regional disaggregate fluctuations. It has developed new tools for modelling dynamically evolving, nonstationary distributions, and applied them to a study of US business cycles.

Section 2 presented a simplistic, naive model. The aim here was to provide a framework for relating distribution dynamics to an explicit economic model. That model served a further concrete role: it highlighted where the investigator needed more empirical facts before proceeding to further theoretical reasoning. As a by-product, however, the model also showed why certain apparently natural point-in-time statistics of cross-section distributions need not be related to aggregate fluctuations, sectoral adjustment, or regional mobility.

The distance between Section 2's theoretical analysis and the empirics in subsequent sections is large but, given available econometric tools, inevitable. The empirical analysis above cannot be interpreted as a test of any assertion from Section 2. Rather, it simply filled in groundwork—future analysis should take the investigation further.

What, however, are the substantive empirical findings at this preliminary stage? Disaggregate dynamics show interesting properties—the mobility indexes and quantile sets are not trivially constant series—but only very particular parts of those disaggregate dynamics are strongly related to aggregate fluctuations. The leading state, varying in identity over time, contains strong predictive power for aggregate fluctuations. By contrast, no single state does so. Why this should be is unclear, but is difficult to understand in models where only aggregate disturbances affect aggregate business cycles through aggregate propagation mechanisms. Instead a better picture might be one of a "wave" of regional dynamics, rippling across the national economy; its initial source varies, depending on which state is the leading one.

Theoretical work to formalize this, and sharpen the empirical analysis, is the next step in this research. One potential way forward is to build on the insights on spillovers and dynamics in Durlauf [8].

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Technical Appendix

This technical appendix contains the proof of Section 2's Proposition.

Proof of Proposition: *Suppose that for some positive number ψ there is a function $\widehat{l}$ in $\mathbb{M}_+$ giving simultaneously:*

(a) For x in $\mathbb{X}$ (a.e.-ϕ_x), $\widehat{l}(x)$ in $\arg\sup_{\lambda \geq 0} \{f(\lambda, z(x)) - \psi\lambda\}$*; and*
(b) $\int \widehat{l}(x)\, \phi_x(dx) = 1$.

Let l be any other element of $\mathbb{M}_+$ such that $\int l(x)\, \phi_x(dx) \leq 1$. By (a) we have for x in $\mathbb{X}$ (a.e.-ϕ_x):

$$f(\widehat{l}(x), z(x)) - \psi \widehat{l}(x) \geq f(l(x), z(x)) - \psi l(x)$$

or

$$f(\widehat{l}(x), z(x)) - f(l(x), z(x)) \geq \psi \left[\widehat{l}(x) - l(x)\right].$$

Integrating with respect to ϕ_x on both sides gives:

$$\begin{aligned} \int \Big(f(\widehat{l}(x), z(x)) - f(l(x), z(x)) \Big)\, \phi_x(dx) &\geq \psi \left[\int \Big(\widehat{l}(x) - l(x) \Big)\, \phi_x(dx) \right] \\ &\geq 0, \end{aligned}$$

so that

$$\int f(\widehat{l}(x), z(x))\, \phi_x(dx) \geq \int f(l(x), z(x))\, \phi_x(dx).$$

Thus, (a) and (b) suffice for $\widehat{l}$ to solve the maximization program. But under the hypotheses of the Proposition, (a) and (i) are equivalent. *Q.E.D.*

Table: Exclusion restriction (Granger causality) tests†
Marginal Significance Levels

	Right hand side bloc		
Left-hand-side Variable	GNP growth	median	maximum
GNP growth	(0.07,0.33)	(0.95,0.47)	(0.03,0.04)
median	(0.23,0.40)	(0.00,0.00)	(0.50,0.44)
maximum	(0.06,0.17)	(0.02,0.28)	(0.00,0.00)

† The first number in each cell entry is the marginal significance level for excluding a right hand side bloc in a 2-lag VAR; the second, in a 5-lag VAR. Systems with lag lengths 3 and 4, as expected, give something in between. All VARs include a constant, and were estimated using annual data from 1948 through 1990.

Figure 4.1
Relative Per Capita Personal Incomes (Log), 1948–1990

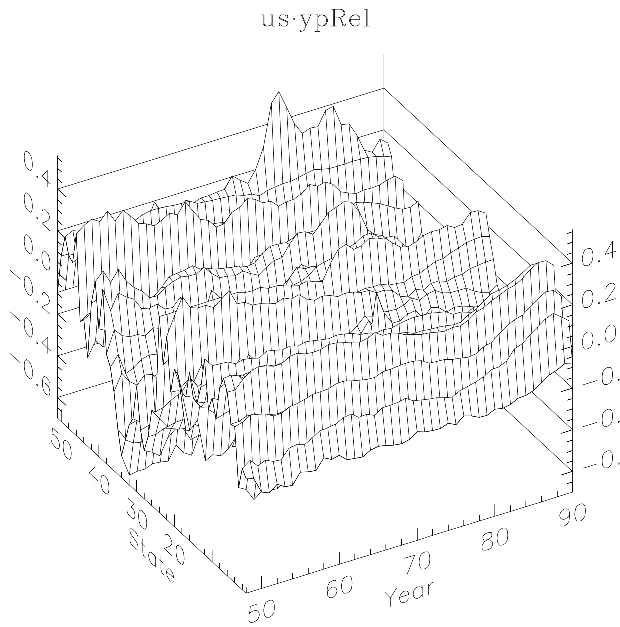

Figure 4.2:
Aggregate and Disaggregate Distribution Fluctuations

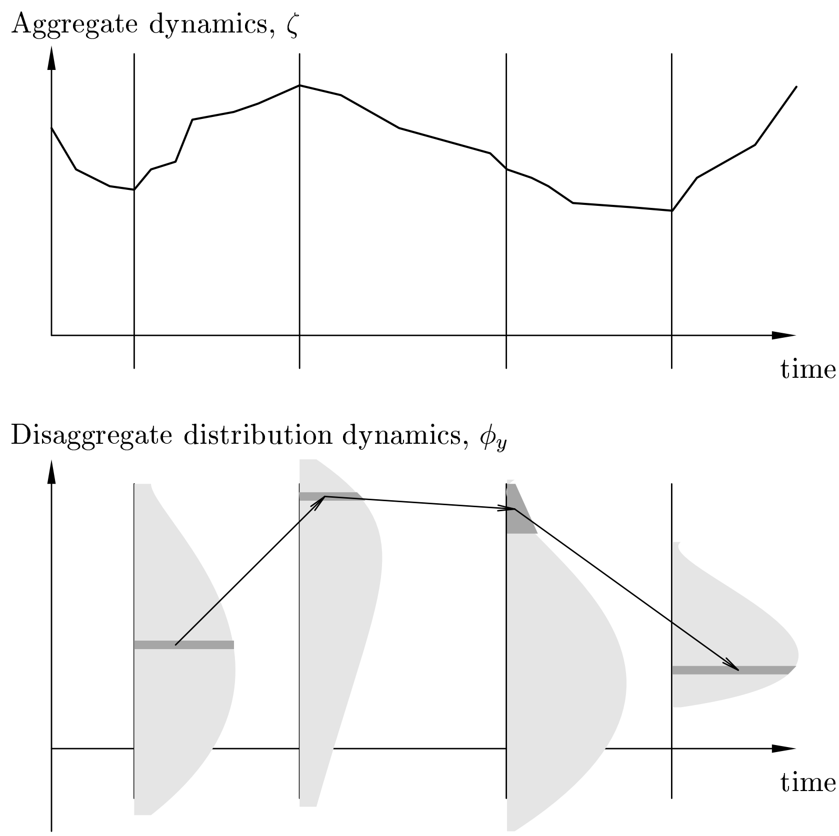

Figure 4.3

Aggregate fluctuations and intra-distribution mobility

In order, GNP growth rates, μ_1, μ_2, μ_3, and μ_{AR}

(See text for definitions.)